Übungen ART II SoSe 2015

Exercise 25 *Observations in Kerr Geometry*

a) Compute the angular velocity of a static observer in the Kerr geometry. The four-velocity of a static observer is assumed to be given by the normal vector of surfaces of fixed Killing time; $t = const$. Compute the angular velocity given by $d\phi/dt$. Furthermore, give the value of the angular velocity on the horizon of the Kerr geometry.

b) Consider the Kerr geometry for small negative r, t fixed and $\theta = \pi/2$. Show that the metric admits closed time like curves.

Exercise 26 *Frame dragging*

Consider the general form of the Kerr metric,

$$ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2 + g_{t\phi}(dtd\phi + d\phi dt)\,, \tag{1}$$

and assume that the metric components are t and ϕ independent with Killing vectors ∂_t and ∂_ϕ.

a) Give the conserved charges for a pointlike test particle moving in the geometry corresponding to energy and angular momentum.

b) Assume the particle is falling from spatial infinity with vanishing angular momentum. Give the condition for zero angular momentum for an asymtotically Minkowski metric $g_{t\phi} \to 0, g_{\phi,\phi} \to 1$ for $r \to \infty$.

c) Compute the position dependent angular velocity $\omega(r, \theta) = d\phi/dt$ for a particle falling towards the black hole from the region with large radial coordinate.