Quantum Field Theory II – WS16 Due 20.1.2017

Exercises – Set 10

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1 Running masses

Similarly as studied for coupling constants, masses in quantum field theories depend on the renormalization scale μ. Their running is governed by the mass renormalization group coefficient, γ_m, through the differential equation:

$$\mu \frac{\partial m(\mu)}{\partial \mu} = -\gamma_m m(\mu) \ . \tag{1}$$

Show that γ_m can be computed from the mass renormalization constant Z_m, according to:

$$\gamma_m = \frac{\partial \log Z_m}{\partial \log \mu} \ . \tag{2}$$

2 Purely gluonic branchings

Derive the expression for the unregularized unpolarized Altarelli-Parisi splitting function:

$$\tilde{P}_{gg}(z) = C_A \left(\frac{1-z}{z} + \frac{z}{1-z} + z(1-z) \right) \ . \tag{3}$$

As a hint, as the three gluons are almost on the mass-shell, take their polarization vectors as purely transverse, contained either on the splitting plane or transverse to it. The unpolarized expression can be obtained from proper sums and averaging of polarized results.

3 DGLAP equation and PDF sum rules

Show that the evolution equation for the PDF of a quark q (and similarly for the anti-quark $\bar{q}$):

$$\frac{df_q(x,Q^2)}{d\log Q^2} = \sum_{i=q,g} \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_{i\to q}(z) f_i\left(\frac{x}{z}, Q^2\right) \tag{4}$$

preserves the corresponding PDF sum rule for the flavor q.