Quantum Field Theory II – WS16 Due 18.11.2016

Exercises – Set 4

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1 Correlation functions

Consider the scalar theory,

$$\mathcal{L} \;=\; \frac{1}{2}\partial_\mu\phi(x)\partial^\mu\phi(x) - \frac{1}{2}m^2\phi^2(x)\,. \tag{1}$$

Compute the two-point function $\langle 0|\phi(x)\phi(y)|0\rangle/\langle 0|0\rangle$ from the path integral. Start with the insertion of a single field and assume that the total derivative,

$$\int \prod_x d\phi(x)\frac{\delta}{\delta\phi(y)}\left\{f[\phi]\exp(iI[\phi])\right\} = 0\,, \tag{2}$$

vanishes.

Next consider the theory with the Lagrangian,

$$\mathcal{L} \;=\; \frac{1}{2}\partial_\mu\phi(x)\partial^\mu\phi(x) - \frac{1}{2}m^2\phi^2(x) + J(x)\phi(x)\,, \tag{3}$$

for which the path integral is given by,

$$Z[J] \;=\; \exp\left[-\frac{i}{2}\int d^4x d^4x' J(x)\Delta(x-x')J(x')\right] Z[0]\,. \tag{4}$$

Compute the two-point function from functional derivatives $(\delta/\delta J(y))$ with respect to the currents. Determine the form of $\Delta(x-x')$ by relating it to the two point function.

2 BRST quantisation – Hilbert space

Consider pure QED with the gauge-fixing function,

$$f = \partial^\mu A_\mu\,. \tag{5}$$

(a) Setup the gauge fixed BRST invariant action using the auxiliary fields ω^*, ω and h.

(b) Give the BRST transformations $[Q,\Phi_A(x)]_\pm = is\Phi_A(x)$ of the fields with Q the BRST charge. Use the equations of motion for the Nakanishi-Lautrup field h and give the on-shell version of the BRST transformations.

(c) Starting from the mode expansions,

$$A^\mu(x) \;=\; \frac{1}{(2\pi)^{3/2}}\int\frac{d^3p}{\sqrt{2p^0}}\left[a^\mu(p)e^{-ipx} + a^{\mu *}(p)e^{ipx}\right]\,, \tag{6}$$

$$\omega(x) \;=\; \frac{1}{(2\pi)^{3/2}}\int\frac{d^3p}{\sqrt{2p^0}}\left[c(p)e^{-ipx} + c^*(p)e^{ipx}\right]\,, \tag{7}$$

$$\omega^*(x) \;=\; \frac{1}{(2\pi)^{3/2}}\int\frac{d^3p}{\sqrt{2p^0}}\left[b(p)e^{-ipx} + b^*(p)e^{ipx}\right]\,. \tag{8}$$

give the transformations of the field modes $[Q,\alpha_A(p)]_\pm$ and $[Q,\alpha_A^*(p)]_\pm$.

(d) Assume a BRST-invariant vacuum $Q|0\rangle = 0$. Under which conditions are the states $a^{\mu *}(p) e_\mu |0\rangle$ physical.

(e) Show that the BRST exact state $Qb^*(p)|0\rangle$ corresponds to the longitudinal polarized states.

(f) Given the commutator relations $[a_\mu(p), a_\nu^*(p')] = -\eta_{\mu\nu}\delta^3(p-p')$ compute the norm of the states $a_0^*(p)|0\rangle$. Should we be worried?

3 Dimensional regularisation

Compute the volume of an $(D-1)$-dimensional sphere of unit radius by analysing the value of the Gaussian integral,

$$\int d^D x e^{-(\vec{x})^2} . \tag{9}$$

Compare the computation in polar coordinates to the computation with the integral being written as a product of lower-dimensional Gaussian integrals.