Quantum Field Theory II – WS16 Due 25.11.2016

Exercises – Set 5

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1 Green functions

Consider an interacting scalar field theory. Use the relation between the generating functional of Green functions and the connected Green functions,

$$Z[J] = \exp\left(iW[J]\right) \tag{1}$$

to relate the four point functions $G(x_1, x_2, x_3, x_4)$ and $G_c(x_1, x_2, x_3, x_4)$. Draw the corresponding diagrammatic representation of the relation.

2 Effective action of free scalar fields

Consider the free massive scalar field theory. Give the generating functional $Z[J]$ and the generating functional of the connected Green functions. Furthermore, derive the effective action $\Gamma[\phi]$ and verify that it is given by the classical action.

3 Vertex functions

Consider pure Yang-Mills theory with a generic gauge group. Compute the vertex function $\Gamma^{abc}_{0,\mu\nu\rho}(x_1, x_2, x_3)$ to leading order in $\hbar$ using,

$$\Gamma^{abc}_{0,\mu\nu\rho}(x_1, x_2, x_3) = \left.\frac{\delta^3\Gamma_0}{\delta A^{a\mu}(x_1)\delta A^{b\nu}(x_2)\delta A^{c\rho}(x_3)}\right|_{\Phi_A=0} . \tag{2}$$

4 Ward identities

Consider QED with the gauge-fixed Lagrangian,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \bar{\psi}[i\gamma^\mu(\partial_\mu - A_\mu) - m]\psi + J^\mu A_\mu - \bar{\psi}J_{\bar{\psi}} + J_\psi\psi \,. \tag{3}$$

(The ghost fields have been integrated out.) Consider the coordinate transformation in the path integral given by the infinitesimal shifts,

$$A_\mu \;\to\; A_\mu + \partial_\mu\delta\theta \,, \quad \psi \;\to\; \psi + ie\psi\delta\theta \,, \quad \bar{\psi} \;\to\; \bar{\psi} - ie\bar{\psi}\delta\theta \,, \tag{4}$$

and assume that the integration measure is invariant.

Use that the path integral is invariant under the field transformations and show the relation,

$$0 = \int D\mathbf{A}D\psi D\bar{\psi}\; \exp\left(i\int dx^4\mathcal{L}\right)\delta\left(\int d^4x\mathcal{L}\right) . \tag{5}$$

Expand the equation (Ward identity) to leading order in the position dependent transformation parameter $\delta\theta(x)$. What relation is implied for the correlation functions for non-vanishing background currents?

Next, transcribe the relation to an equation in terms of functional derivatives of the generating functionals of the Green functions.

Finally, transcribe the relation to an equation for the generating functional of the connected Green functions $W[J]$ and the effective action $\Gamma[\phi]$.