Quantum Field Theory II – WS16 Due 16.12.2016

Exercises – Set 7

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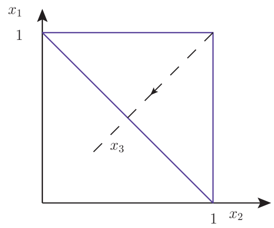
1 Phase space in D dimensions

We found that the cross section for three-parton production at an e^+e^- machine is associated with the phase-space integral

$$\int dx_1 dx_2 \frac{x_1^2 + x_2^2}{(1-x_1)^{\nu_1}(1-x_2)^{\nu_2}} \;, \tag{1}$$

with $\nu_1 = \nu_2 = 1$, potentially covering the integration region defined by the triangle in:



We want to regularize the divergences that appear at the boundaries $x_1 = 1$ and $x_2 = 1$.

(a) Assume $\nu_1 = \nu_2 = 1 - \delta$, with δ real and positive and compute the modified phase-space integral. Write down explicitly the poles in δ that you obtain for $\delta \to 0$.

(b) We will find that by keeping the dimension of space-time as a parameter $D = 4 - 2\epsilon$, one can achieve the previous regularization procedure. Indeed, using:

$$d^{(D)}\text{LIPS}_3 = \frac{2^{2\epsilon}}{32(2\pi)^{5-4\epsilon}} s^{1-2\epsilon} d\Omega_{D-2} d\Omega_{D-3} \frac{dx_1 dx_2}{(1-x_1)^\epsilon (1-x_2)^\epsilon (x_1+x_2-1)^\epsilon} \tag{2}$$

and the D-dimensional expression for the integrand

$$x_1^2 + x_2^2 \to (x_1^2 + x_2^2)(1-\epsilon) + 2\epsilon(x_1 + x_2 - 1) \;,$$

compute the total real contributions to 2-jet production at e^+e^- colliders.

(c) Express your result as a Laurent series around $\epsilon = 0$, factoring out the constant:

$$\frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)^2}{\Gamma(1-3\epsilon)} \left(\frac{s}{4\pi\mu^2}\right)^{-\epsilon} \tag{3}$$

where we have introduced the *regularization scale* μ^2 in order to keep proper dimensions for the total cross section.

2 Tensor integrals for 3-point vertex

(a) Write down in full the $\mathcal{O}(\alpha_s)$, 1-loop integral expression for three-point vertex $\gamma^* \to q\bar{q}$.

(b) One of the tensor integrals that contributes to the three point vertex, has the structure:

$$\int \frac{d^D t}{(2\pi)^D} \frac{t^\mu t^\nu}{t^2 (t-q_1)^2 (t-q_2)^2} \ . \tag{4}$$

Express this integral as a linear combination of the metric tensor and a proper tensor built up from the vectors q_1^μ and q_2^μ.

(c) Show that the previous tensor integral, when fully contracted with the metric tensor, can be expressed in terms of a lower point integral. This implies a non-trivial relation for the decomposition written in the previous item.