Quantum Field Theory II – WS16 Due 11.11.2016

Exercises – Set 3

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1 Generating functions

Consider the integral of a 'zero' dimensional field theory,

$$Z(\lambda, j, n) = \int d\phi\, \phi^n \exp(\mathcal{L}), \quad \mathcal{L} = -\left[\frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4\right] - j\phi, \tag{1}$$

where j is an arbitrary source term for the field ϕ.

(a) Compute the integral for the free theory including the source term but without field insertions ($n = 0$).

(b) Compute the two point function $Z(\lambda = 0, j = 0, n = 2)$ to leading order in λ.

(c) Compute the four point function to leading order in λ and vanishing current: $j = 0$.

(d) Compute the four-point function to first order in λ.

2 BRST transformations

Given is the form of the BRST transformations $s\Phi$ for a Yang-Mills theory,

$$sA_{a\mu} := \partial_\mu\omega_a + C_{abc}A_{b\mu}\omega_c, \quad s\omega^*_a := -h_a, \tag{2}$$

$$s\omega_a := -\frac{1}{2}C_{abc}\omega_b\omega_c, \quad sh_a := 0, \tag{3}$$

where the fields ω and ω^* are Grassmann valued. The generalised product rule applies to the transformations,

$$s(\Phi_1\Phi_2) = (s\Phi_1)\Phi_2 \pm \Phi_1(s\Phi_2). \tag{4}$$

The lower sign holds for Φ_1 being Grassmann valued. Show that the transformations are nilpotent by verifying $s^2(\Phi) = 0$ for all fields.

3 Canonical formalism

Consider a massive vector field with the Lagrangian,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2V_\mu V^\mu + J_\mu V^\mu, \quad F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu. \tag{5}$$

Compute the constraints ($\chi_{1,2;x}$) and their Poisson brackets of the system for the massive ($m \neq 0$) and the massless ($m = 0$) theory:

(a) Compute the canonical momenta and give the primary constraint $\chi_{1;x}$.

(b) The secondary constraint is obtained from the field equation of V^0. Compute the constraint $\chi_{2;x}$.

(c) Give the form of the Poisson brackets for fields using standard notation (making integrations explicit). Start from the the known Poisson bracket formula,

$$\{A, B\} = \frac{\partial A}{\partial Q^a}\frac{\partial B}{\partial \Pi_a} - \frac{\partial B}{\partial Q^a}\frac{\partial A}{\partial \Pi_a}\,, \tag{6}$$

with all canonical variables evaluated at equal times.

(d) Compute the Poisson brackets of the constraints. Show that $\chi_{i;x}$ are first class constraints for vanishing mass parameter.

(e) Show that the time-derivative of the secondary constraint is proportional to the field equations. Are there tertiary constraints?