Quantum Field Theory II – WS16 Due 21.12.2016

Exercises – Set 8

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1 Feynman Parameters

One way of computing loop-momentum integrals in quantum field theory is by using the so called *Feynman parameterization*.

(a) Show the relation:

$$\frac{1}{D_1 D_2} = \int_0^1 dx_1 dx_2 \delta(1 - x_1 - x_2) \frac{1}{(x_1 D_1 + x_2 D_2)^2} \;, \tag{1}$$

where D_1 and D_2 are independent of x_1 and x_2 and the integration region is defined by $0 < x_i < 1$ ($i = 1, 2$).

(b) By differentiation, using the previous expression, show:

$$\frac{1}{D_1 (D_2)^m} = \int_0^1 dx_1 dx_2 \delta(1 - x_1 - x_2) \frac{m x_2^{m-1}}{(x_1 D_1 + x_2 D_2)^{m+1}} \;. \tag{2}$$

(c) With the previous results, prove by induction the so called *Feynman parameterization*:

$$\frac{1}{D_1 D_2 \cdots D_n} = \int_0^1 dx_1 \cdots dx_n \delta(1 - x_1 - \cdots - x_n) \frac{(n-1)!}{(x_1 D_1 + \cdots + x_n D_n)^n} \;, \tag{3}$$

in which the integration volume is defined by $0 < x_i < 1$ for all i's.

2 Massless two-point scalar integral

Consider the following "bubble" 1-loop integral:

$$B_0(q^2) = \frac{(2\pi\mu)^{(4-D)}}{i\pi^2} \int \frac{d^D t}{(t^2 + i\varepsilon)((t+q)^2 + i\varepsilon)} \;. \tag{4}$$

(a) Use Feynman parameters to show the relation:

$$B_0(q^2) = \frac{(2\pi\mu)^{(4-D)}}{i\pi^2} \int_0^1 dx \int \frac{d^D t}{g(t, q, x)^2} \;, \tag{5}$$

with $g(t, q, x) = t^2 + 2x(t \cdot q) + xq^2$.

(b) Make a change of variables, $t' = t + xq$, to express $g(t, q, x) = (t')^2 - H(x, q^2)$, in which $H(x, q^2) = -q^2 x(1 - x) - i\varepsilon$.

(c) Use the tadpole integral:

$$\int \frac{d^D t}{(2\pi)^D} \frac{1}{(t^2 - \Delta + \mathrm{i}\varepsilon)^n} = \frac{(-1)^n i}{(4\pi)^{D/2}} \frac{\Gamma(n - D/2)}{\Gamma(n)} \left(\frac{1}{\Delta - \mathrm{i}\varepsilon} \right)^{n - D/2} \;, \tag{6}$$

to get the relation ($D = 4 - 2\epsilon$):

$$B_0(q^2) = (4\pi)^{\epsilon} \Gamma(\epsilon) \int_0^1 dx \left(\frac{\mu^2}{H(x, q^2)} \right)^{\epsilon} \;. \tag{7}$$

(d) Expand the integrand in ϵ, in order to show:

$$B_0(q^2) = (4\pi)^\epsilon \left(\frac{1}{\epsilon} + 2 - \ln\left(\frac{-q^2 - i\varepsilon}{\mu^2} \right) + \mathcal{O}(\epsilon) \right) \, . \tag{8}$$