Quantum Field Theory II – WS16   Due 3.2.2017

Exercises – Set 12

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1   Goldstone bosons and broken symmetries

Consider the Goldstone model with a complex scalar field $\phi(x)$ and the potential $V(\phi) = \lambda/4 \left(|\phi|^2 - \mu^2/\lambda \right)^2$.

(a) In the unbroken phase ($\mu^2 < 0$), compute the Noether current $j^\mu(x)$ associated with the global symmetry $\phi \to e^{i\alpha}\phi$, $\phi^* \to e^{-i\alpha}\phi^*$.

(b) Using the Kibble parameterization:

$$\phi(x) = \exp\left(i\frac{\xi(x)}{v} \right) (v + \rho(x)) , \tag{1}$$

with $v = \sqrt{\mu^2/\lambda}$, show that after spontaneous symmetry breaking (with $\mu^2 > 0$), the $\xi(x)$ dependent terms in the Lagrangian, can be written as:

$$-\frac{1}{v}\xi(x)\partial_\mu j^\mu(x) . \tag{2}$$

2   Details from the lecture's proofs

(a) Show that a general complex square matrix Y can be decomposed as $Y = U^\dagger M V$, with U and V unitary matrices and M a diagonal matrix with zero or positive entries in the diagonal.

(b) Show that a general complex multiplet of fields ψ with dimension n, transforming under a complex representation of a symmetry group G can be mapped into a $2n$-dimensional real multiplet ϕ transforming under a real representation of G. First give the transformation matrices of the real and imaginary part of the fields. Furthermore, show that the representation matrices obey the commutator relations of the Lie algebra.

3   Massive neutrinos

By the measurement of neutrino oscillations, we know that neutrinos are massive, though their masses are much smaller than any other massive particle in the Standard Model. For the following assume that right-handed neutrinos are added to the lepton sector of the Standard Model in order to mimic the mass-generation mechanism of the quarks.

(a) What quantum numbers have to be assigned to a right-handed neutrino ν_R in order to write Yukawa interactions with the Higgs field which is analogue to the one for the quarks?

(b) Start with a generic mixing matrix and use field redefinitions that keep the kinetic terms diagonal in order to identify the independent parameters.