Quantum Field Theory II – WS16 Due 9.12.2016

Exercises – Set 6

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1 Counter term Lagrangian of QCD

Consider the bare Lagrangian of QCD including gauge fixing terms,

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^{a\mu})^2 + \partial^\mu \bar{c}^a(\delta^{ac}\partial_\mu + gf^{abc}A^b_\mu)c^c \quad (1)$$

$$+\bar{\psi}_i(\gamma^\mu(i\delta_{ij}\partial_\mu + gA^a_\mu T^a_{ij}) - m\delta_{ij})\psi_j \,. \quad (2)$$

Introduce renormalized fields and couplings and give the counter term Lagrangian.

Give the Feynman rules for the theory by collecting the propagator diagrams, and vertex diagrams including the counter term diagrams.

2 On-shell renormalization conditions

Revisit the on-shell renormalization conditions for a massive lepton, introduced in the lectures. Derive the two renormalization conditions for the scalar and vectorial part of the two-point vertex functions.

Next, consider an interacting, massive scalar field and give its renormalization conditions in the on-shell scheme. How many renormalization constants are expected? Give the counter terms that are quadratic in the fields.

3 Bare photon vacuum polarization contribution

Start from the bare vacuum polarization contribution and derive the singular part by expanding the result around four dimensions, i.e. $\delta \to 0$. Derive the divergent part. Furthermore, derive the value of the counter term δZ_A in the on-shell renormalization scheme.

4 Self-energy corrections

Consider the one-loop self energy corrections of the photon and the electron.

(a) Draw the Feynman diagrams and use the Feynman rues to obtain the Feynman amplitudes. (It is sufficient not to perform the integrals.)

(b) Next consider the contributions to the gluon and quark self energies and draw the Feynman diagrams.

(c) Finally, work out the color algebra and compare the results to the previously ones obtained for QED.