Quantum Field Theory II – WS16 Due 13.1.2017

Exercises – Set 9

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1 Superficial Degree of Divergence in QED

While studying renormalized perturbation theory, we introduced the concept of *superficial degree of divergence* ($\omega(G)$) for a one-particle-irreducible (1PI) graph (G) in order to study UV properties of quantum field theories. We deduce now an expression for $\omega(G)$ in QED in d dimensions.

(a) Consider a 1PI graph G in QED with L loops and I_ψ and I_A propagators for fermions and photons respectively. By exploring the leading UV contributions to loop-momentum integrations, find a relation for $\omega(G)$ as a function of d, L, I_ψ and I_A.

(b) Show the *Euler-loop equation*: $L = I_\psi + I_A - V + 1$, where V is the number of vertices in G.

(c) Employing the number of external fermions E_ψ and photons E_A in G, show the relations $V = I_\psi + 1/2 E_\psi$ and $V = 2I_A + E_A$.

(d) With these relations, show that $\omega(G)$ can be written in terms of only d, V, E_ψ and E_A.

(e) As a function of d, is QED renormalizable?

2 QED one-loop 1- and 3-point vertex functions

(a) Compute the superficial degree of divergence for the 1- and 3-photon vertex functions Γ^A and Γ^{AAA}.

(b) Show by explicit calculation, that at one loop Γ^A vanishes.

(c) Draw the 1-loop diagrams that contribute to Γ^{AAA}, and show that they add to zero.

3 Regularized splitting functions

We computed explicitly the *unregularized* splitting function $\tilde{P}_{qq}(z) = C_F(1+z^2)/(1-z)$, where an explicit divergence, of *soft* nature, remained at $z = 1$. By adding the corresponding 1-loop contributions this divergence cancels, such that the full result is written in terms of the *regularized* splitting function in 4 dimensions as $P_{qq}(z) = C_F\left((1+z^2)/(1-z)\right)_+$. We have employed the *plus*-distribution defined as:

$$\int_0^1 dz F(z)_+ h(z) = \int_0^1 dz (F(z)h(z) - F(z)h(1)) \; . \tag{1}$$

Show that $P_{qq}(z)$ can be written as $C_F((1+z^2)\,(1/(1-z))_+ + 3/2\delta(1-z))$ (hint: you can use the relation $F(z)_+ = F(z) - \delta(1-z)\int_0^1 dy F(y)$).