Exercises for QFTI **SS 2017**

Exercise 1 *Properties of the Schrödinger Equation* (3 points)

a) Consider Schrödinger's equation,

$$\left(i\frac{\partial}{\partial t}+\frac{1}{2m}\left(\nabla_x\right)^2\right)\psi(t,\vec{x})=0\,. \tag{1}$$

Find a Lagrangian that yields Schrödinger's equation as its Euler-Lagrange equation. (Hint: Consider the real and the imaginary wave function ψ and ψ^* as independent fields.)

b) Show that the Lagrangian is invariant under the following transformation,

$$\psi\to e^{-i\alpha}\psi\,,\quad \psi^*\to e^{i\alpha}\psi^*\,, \tag{2}$$

and compute the associated Noether current $\{J^t,\vec{J}\}$. What is the interpretation of the associated conserved charge?

c) Compute the energy-momentum tensor and the associated conserved charges, $\{E,\vec{P}\}$.

Exercise 2 *Euler-Lagrange Equations (S)* (2 points)

Derive the generalization of the Euler-Lagrange equations for general Lagrangians of the form $\mathcal{L}[\phi,\partial_\mu\phi,\partial_\mu\partial_\nu\phi]$.

Exercise 3 *Ambiguities in the Energy-Momentum Tensor (S)* (3 points)

a) If you add a total derivative to a Lagrangian $\mathcal{L}(\phi,\partial_\nu\phi)\to\mathcal{L}(\phi,\partial_\nu\phi)+\partial_\mu X^\mu(\phi,\partial_\nu\phi)$, how do the equations of motion change? How does the energy-momentum tensor change?

b) Show that the total energy $E=\int T^{00}d^3x$ is invariant under such changes.

c) Show that $T^{\mu\nu}\neq T^{\nu\mu}$ is not symmetric for $\mathcal{L}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$. Find an X^μ so that $T^{\mu\nu}$ is symmetric in this case?