Exercises for QFTI SS 2017

Complete exercises 9 and 10 from the previous exercise sheet.

Exercise 11 *Two-to-two scattering (see e.g. Schwartz section 5.1.2)* (3 points)

Consider the kinematics of a $2 \to 2$ scattering process. Assume that the colliding particles move in opposite direction along the z axis and that the sum of the initial-state momenta is given by $P = \{E_{cm}, 0, 0, 0\}$. The final state particles have generic masses $m_{3,4}$.

a) Solve the on-shell conditions $p_i^2 = m_i^2$ and momentum-conservation condition $P = p_3 + p_4$ using a parametrisation in terms of polar and azimuthal angle between the final state three momentum $\vec{p}_3$ and the z-axis. How many independent momentum components are expected? Draw the mass-shell conditions and the allowed final-state momenta in the $p_f^x = 0$ slice of momentum space.

b) Show that the Lorentz invariant phases-space measure,

$$d\Pi_{2,\text{LIPS}} = (2\pi)^4 \delta^4(P - p_3 - p_4) \prod_{f=3,4} \frac{d^3 p_f}{(2\pi)^3} \frac{1}{2E_f}\,, \tag{1}$$

is given by,

$$d\Pi_{2,\text{LIPS}} = d\Omega \frac{1}{16\pi^2} \frac{|\vec{p}_3|}{E_{\text{cm}}} \theta(E_{\text{cm}} - m_3 - m_4)\,, \tag{2}$$

where $d\Omega = d\phi d(\cos\theta)$ denotes the angular volume element associated to polar and azimuthal angles between the final state three momenta. What is the physical interpretation of the θ-function?

c) Show that the cross section for such a process is given by,

$$d\sigma = d\Omega \, \frac{1}{64\pi^2 E_{\text{cm}}^2} \frac{|\vec{p}_3|}{|\vec{p}_1|} |\mathcal{M}|^2 \theta(E_{\text{cm}} - m_3 - m_4)\,. \tag{3}$$