Exercises for QFTI **SS 2017**

Exercise 12 *Wick theorem* (5 points)

Consider the time-ordered operator product for the real, free Klein-Gordon field theory and show the relation,

$$T\{\phi(x_1)\phi(x_2))\} =: \phi(x_1)\phi(x_2) + D_F(x_1, x_2) : \tag{1}$$

following the below steps. In the following ϕ denotes the free field.

a) Are the vacuum expectation values of all normal-ordered expressions zero? What about the vacuum expectation values of an odd number of operators?

b) Do you know quantum fields of nature with non-vanishing expectation value? Give at least two examples!

c) Split the field operators into positive and negative frequency modes,

$$\phi(x) = \phi_+(x) + \phi_-(x)\,, \tag{2}$$

$$\phi_+(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} a^\dagger e^{ixp}\,, \quad \phi_-(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} a e^{-ixp}, \tag{3}$$

and show the above relation (1). To this end start with the product of two operators and reorganise the expression into normal-ordered terms and a remainder. Show that the remainder is in fact the Feynman propagator. (2 point)

d) Argue that the correlator of an odd number of fields gives zero for the free theory we discuss here. Is the product of two time-ordered expressions time-ordered? What about the product of two normal ordered expressions?

Exercise 13 *Dyson series* (3 points)

Consider the evolution operator $U(t, t_0)$ for the free, real Klein-Gordon field theory with

$$i\frac{\partial}{\partial t} U(t, t_0) = H_{\text{int}}(t) U(t, t_0)\,. \tag{4}$$

a) Use the integrated differential equation,

$$U(t, t_0) = 1 + (-i) \int_{t_0}^{t} dt' H_{\text{int}}(t') U(t', t_0)\,, \tag{5}$$

to show that a solution for $U(t, t_0)$ is given by the Dyson series,

$$U(t, t_0) = 1 + (-i) \int_{t_0}^{t} dt' H_{\text{int}}(t') + (-i)^2 \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' H_{\text{int}}(t') H_{\text{int}}(t'') + \cdots \tag{6}$$

Is this expression time ordered?

b) Show the below multiplication rules of the evolution operator $U(t, t')$,

$$U(t_1, t_2)U(t_2, t_3) = U(t_1, t_3)\,, \quad U(t_1, t_2)(U(t_3, t_2))^\dagger = U(t_1, t_3)\,. \tag{7}$$

for $t_1 > t_2$ and $t_3 > t_2$.

c) Is $(U(t_3, t_2))^\dagger$ as introduced in question (c) time ordered? Is the expression $U(t_1, t_0)(U(t_3, t_0))^\dagger = U(t_1, t_3)$ time ordered? (Assume $t_1, t_3 > t_0$ and $t_1 > t_3$.)