Exercises for QFTI **SS 2017**

Please hand in your exercises before Wednesday 14.6.2017. There will be a lecture instead of the tutorial on Tuesday 13.6.2017. We will hand out a solution sheet and mark your exercises.

Exercise 15 *Probability conservation in QFT* (4 points)

This formal exercise aims to give an intuition about the conserved charge associated to probability in QFT. Where does it come from? Does it depend on the field theory we quantise?

a) Consider a simple quantum mechanical system, e.g. free particle $\vec{x}(t)$ moving in a potential well. What symmetry transformation on the wave function $\psi[\vec{x}(t_0); t]$ is associated to probability, once the system is quantised? Discuss the symmetry at the level of the Schoedinger equation and for the associated Lagrangian. Give the conserved current. (You can reuse the results from the earlier exercises.)

b) Consider a real scalar quantum-field theory. By analogy the wave function is given by the functional $\psi[\phi(\vec{x}, t_0), t]$. Give a differential operator that corresponds to the momentum operator conjugate to the field variables $\phi(x, t_0)$ at a fixed time t_0.

c) Assume the following Hamiltonian,

$$H[\phi] = \int d^3x \left[\frac{1}{2}\pi(x, t_0)^2 + \frac{1}{2}\partial_i\phi(x, t_0)\partial_i\phi(x, t_0) + V[\phi(\vec{x}, t_0)] \right] . \tag{1}$$

Give the Schroedinger equation. Construct a Lagrangian for the functional $\psi[\phi(\vec{x}, t_0), t]$ that has the Schroedinger equation as an extremum.

d) By analogy to quantum mechanics, give the symmetry transformation associated to probability conservation. What is the associated Noether current, what the conserved charge?

Remark: If in doubt consider fields as a large number of discrete degrees of freedom $\phi(\vec{x}, t) \leftrightarrow \phi_x(t)$ labeled by x. Don't be afraid to take derivatives with respect to the field variable or integrate over them, i.e. use $\partial/\partial\phi_x(t_0)$ or $\int[\prod_x d\phi_x] f[\phi_x(t)]$.

Exercise 16 *Contractions and diagrams* (3 points)

Consider the vacuum expectation value,

$$\langle 0|Texp[-i\int d^4x V_{\rm int}[\phi(x,t)]]|0\rangle \quad V_{\rm int}[\phi(x,t)] := \frac{\lambda}{4!}\phi(x,t)^4 \tag{2}$$

for the free Klein-Gordon field $\phi(x,t)$.

a) State the correlators that have to be computed. Discuss the coupling factors and combinatorial factors for the leading order (LO), next-to-leading (NLO) order and the next-to-next-to-leading order (NNLO) contributions.

b) List the contractions and draw the Feynman diagrams for the NLO contribution. Work out the symmetry factors and explain them from a diagrammatic point of view.

c) Draw the connected diagrams of the NNLO contribution and work out the numerical prefactors. Interpret the prefactors as symmetry factors.