Tutorial 10

February 1, 2016

Quick disclaimer: This is the last scheduled tutorial. If you would like to have an extra session to ask questions before the exam, please discuss between yourselves to agree on a date (between Wednesday 3rd and Friday 5th) and time, and let me know next Monday what you have agreed on. I will hand back and briefly discuss the second test.

Exercise 1

The aim of this exercise is to derive the form of the Feynman propagator for a massive scalar field $\phi(x)$ with mass m,

$$D_F(x_1, x_2) = \langle 0|T\{\phi(x_1)\phi(x_2)\}|0\rangle = -i \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + m^2 - i\epsilon} e^{ikx} \,. \tag{1}$$

Use the form of the free field operator,

$$\phi(x,t) = \int \frac{d^3k}{2E_k(2\pi)^3} \left(a_k e^{-ikx} + a_k^* e^{ikx} \right)$$

as well as the commutator relations,

$$[a_k, a_{k'}^*] = 2E_k \, (2\pi)^3 \, \delta^{(3)}(k - k') \,.$$

1) To start with show the identify,

$$e^{-iw\tau}\theta[\tau] + e^{iw\tau}\theta[-\tau] = \lim_{\epsilon \to 0} \frac{2w}{2\pi i} \int_{-\infty}^{\infty} \frac{d\omega}{-\omega^2 + w^2 - i\epsilon} e^{i\omega\tau} \,. \tag{2}$$

   Notice that ϵ^2-terms can be neglected and one can use $2\epsilon w \to \epsilon$ when considering the limit.

2) Next compute the correlator, $\langle 0|\phi(x_1)\phi(x_2)|0\rangle$ without the time ordering.

3) Finally take the time-ordering and relation (2) into account to obtain the Feynman propagator.

4) What do you obtain when inserting the propagator into the field equation, $(-\partial_\mu \partial^\mu + m^2) D_F(x, 0)$?

Exercise 2

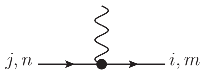
The sector of the Standard Model lagrangian describing the coupling of a W^+ boson with a pair of fermions $f_n\bar{f}_m$ is

$$\mathcal{L} = ie_W W_\mu^+ U_{nm}\bar{f}_m\gamma^\mu(1+\gamma_5)f_n, \tag{3}$$

where

$$U_{nm} = \begin{cases} V_{nm} & \text{if } f_m f_n \text{ are quarks} \\ \delta_{nm} & \text{if } f_m f_n \text{ are leptons} \end{cases}. \tag{4}$$

This implies the vertex $W^+f_n\bar{f}_m$ is given by:



$$= -e_W U_{mn}\left(\gamma^\mu(1+\gamma_5)\right)_{ij}(2\pi)^4\delta^4(k+l+p)\,. \tag{5}$$

By analogy, determine the expression for the vertices $W^-f_n\bar{f}_m$, $Zf\bar{f}$ and $\gamma f\bar{f}$.

Exercise 3

Draw the Feynman diagrams describing the process $e^+e^- \to q\bar{q}$.

1) Given the expression for the vertices $Zf\bar{f}$ and $\gamma f\bar{f}$ you determined in the previous exercise, what is the the Lorentz structure of the vertices?

2) The S-matrix is a Lorentz scalar, which means the Lorentz indices of the vertices must be contracted. This is guaranteed by the fact that the propagator of a spin one particle is not a Lorentz scalar. Given that a propagator can only depend on the properties of the particle it describes, how could you dress eq. (1) so that it has the Lorentz structure necessary to contract the free indices of the vertices?

Exercise 4

Draw the Feynman diagrams and determine the matrix elements for

1) $Z^0 \to f\bar{f}$;

2) $\mu^- \to e^-\bar{\nu}_e\nu_\mu$.