Tutorial 9

January 18, 2016

Exercise 1: Total decay rate of W^+

We first compute the decay rate of $W^+ \to \bar{f}_m f_n$. The relevant sector of the SM Lagrangian is

$$\mathcal{L} = ie_W W^+ V_{nm} \bar{u}_m \gamma^\mu (1+\gamma_5) d_n \,. \tag{1}$$

This has a similar structure to the Lagrangian relevant for the decay $W^- \to e^- \bar{\nu}_e$ we studyied in the previous tutorial, if we set $g_V = g_A = 1$ (as we should have done there) and include the CKM matrix V_{nm}.

Reproducing the steps leading to eq. (9) of `tutorial8Correction.pdf`, show that

$$\left|\mathcal{M}\left(W^+ \to \bar{f}_m f_n\right)\right|^2 = 8e_W^2 \left(2\epsilon \cdot p\, \epsilon \cdot q - p \cdot q\right) |V_{mn}|^2 N_c \,, \tag{2}$$

where $N_c = 3$ is the number of quark colours. In the CM frame of the decaying W^+ boson, for unpolarized bosons and neglecting fermion masses, show that

$$\Gamma\left(W^+ \to \bar{f}_m f_n\right) = \frac{\alpha M_W}{12 \sin^2 \theta_W} N_c |V_{mn}|^2 \,. \tag{3}$$

This is obtained in exactly the same way as we got the unpolarised rate for $W^- \to e^- \bar{\nu}_e$ in tutorial 8, and we recall $e_W = \frac{e}{2\sqrt{2} \sin \theta_W}$ and $\alpha = \frac{e^2}{4\pi}$.

Noting $\Gamma\left(W^+ \to e^+ \nu_e\right) = \Gamma\left(W^- \to e^- \bar{\nu}_e\right)$, show that

$$\Gamma\left(W^+ \to \bar{f}_m f_n\right) = N_c |V_{mn}|^2 \Gamma\left(W^+ \to e^+ \nu_e\right) \,.$$

To compute the total rate, we must sum over all possible final states. Comparing the mass of leptons and quarks with the mass of W^+, which final states are kinematically allowed?

Summing over all possible final states, show that

$$\Gamma_{\text{tot}}(W^+) = \Gamma_{\text{tot}}(W^-) = 9\Gamma\left(W^+ \to e^+ \nu_e\right) = 2.04\,\text{GeV} \,. \tag{4}$$

Hint: Recalling that V_{mn} is unitary, verify $\sum_{n=1}^{2} \sum_{m=1}^{3} |V_{mn}|^2 = 2$.

Compute the lifetime τ of a $W^\pm$ boson. Assuming it travels at the speed of light (and neglecting relativistic effects), can it be directly measured in a detector?

Exercise 2: Decay rate of the Higgs boson into fermions

Starting from

$$\mathcal{L}_H = \sum_f \frac{m_f}{v} H \bar{f} f + 2\frac{M_W^2}{v} H W_\mu^+ W^{-\mu} + \frac{M_Z^2}{v} H Z_\mu Z^\mu \tag{5}$$

follow the usual procedure to show:

- $\mathcal{M}(H \to f\bar{f}) = \left\langle f(p,\sigma)\bar{f}(q,\delta)\right| \mathcal{H}_H \left|H(k)\right\rangle = \frac{m_f}{v}\bar{u}(p,\sigma)v(q,\delta)$;
- $\left|\mathcal{M}(H \to f\bar{f})\right|^2 = -\frac{4m_f^2}{v^2}\left(p\cdot q + m_f^2\right)$;
- in the CM frame of the decaying H boson,

  $$\Gamma(H \to f\bar{f}) = \frac{|p|}{8\pi M_H^2}\left|\mathcal{M}(H \to f\bar{f})\right|^2 \approx \frac{M_H}{8\pi}\left(\frac{m_f}{v}\right)^2 .$$

  where the approximation is valid for $m_f^2 \ll M_H^2$.
  Hint: by symmetry, the energy of the final state particles must be equal to $M_H/2$.

Given that the decaying rate is proportional to the mass squared of the fermion in the final state, the decay rate is largest for the heaviest fermion the Higgs can decay into (recall $M_H = 125$ GeV). What is this fermion? Compute the numerical value of this decay rate, and the corresponding lifetime. Assuming the Higgs boson travels at the speed of light and neglecting relativistic effects, how far does the Higgs boson travel?

Exercise 3: Decay rate of the Higgs boson into W bosons

Starting from the relevant term of eq. (5), compute the decay rate $\Gamma(H \to W^+W^-)$ following the steps:

- $\mathcal{M}(H \to W^+W^-) = \langle W^+(p,\sigma)W^-(q,\delta)|\, \mathcal{H}_H \,|H(k)\rangle = \frac{2M_W^2}{v}\epsilon_\mu^*(p,\sigma)\epsilon^{*\mu}(q,\delta)$;
- using

  $$\sum_\lambda \epsilon_\mu^*(k,\lambda)\epsilon_\nu^*(k,\lambda) = \eta_{\mu\nu} + \frac{k_\mu k_\nu}{M^2} ,$$

  show that $|\mathcal{M}(H \to W^+W^-)|^2 = \frac{4M_W^2}{v^2}\left(2 + \frac{(p\cdot q)^2}{M_W^4}\right)$;
- in the CM frame of the Higgs boson show that $\Gamma(H \to W^+W^-) \approx \frac{M_H^3}{16\pi v^2}$ in the approximation $M_W \ll M_H$. Unlike what happens for the decay rate into fermions, this rate is independent of the mass of the bosons in the final state.

What would be the observed final-state particles in a detector from the decay of a Higgs boson into W bosons? Given the measured mass of the Higgs boson, is this decay channel allowed?